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Houba, H.E.D.; van der Laan, G.; Tieman, A.F.

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Alexander F. Tieman[†] Gerard van der Laan[‡] Harold Houba[§]

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Abstract

We analyze the behavior of producers who compete through price competition in a social environment from a sociological point of view. The standard model of Bertrand price competition is enriched with producers who follow a 'Win Cooperate, Lose Defect' (WCLD) strategy. This strategy is a behavioral rule, based on the notion of fairness or aspirations, and can be regarded as a modified Tit-for-Tat strategy. By letting all producers follow the WCLD strategy an evolutionary process is specified.

The model can explain the emergence of cooperative behavior. The model also shows that occasionally local price wars may occur. At a price war, locally prices decrease, and return to the old, higher price after a stochastic time.

Key-words: *Evolutionary Game Theory, Social Interaction, Local Interaction, Bertrand Price Competition, Evolution of Cooperation, Tit-for-Tat.*

JEL code: C70, D20, L13, L20.

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[†]Tinbergen Institute and Free University. Mail address: Dept. of Econometrics, Free University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email:XTieman@econ.vu.nl. phone: +31-20-4446022, fax: +31-20-4446020. URL: http://www.econ.vu.nl/~xtieman.

[‡]Tinbergen Institute and Free University.

[§]Free University.

I. Introduction.

In this paper we consider the behavior of producers who are involved in price competition on a specific market. We regard competition in a social environment. A social environment consists of a number of agents who can socially interact with each other. According to this description a lot of every-day-life situations can be described as occurring in a social environment. In fact every act of interaction takes place in a social environment with at least two agents. In most of the economic literature on this subject the behavior of agents in a social environment is viewed from a purely economic perspective. Recently, attention has focused on the selection of equilibria by means of following the biological concept of evolution consisting of fitness selection in combination with random mutation (see e.g. Young (1993), Kandori, Mailath, and Rob (1993) and Ellison (1993, 1995)). In the field of evolutionary economics this kind of fitness selection is known as Darwinian dynamics with mutations.

In this paper we consider the problem from a different perspective by introducing agents who adopt sociological reasoning. We specify adaptations on the level of the individual, instead of considering an adjustment process for a group of individuals. We try to establish a link between the individual's behavior and the behavior of the group the individual belongs to as a whole. This is known in psychology as the *level of analysis problem* (see Messick and Liebrand (1995)), and it is addressed in economics by Schelling (1978) and Young (1998). It is the combination of these features that makes our model fundamentally different from most dynamics in evolutionary game theory.

We implement the adjustment process by introducing a decision heuristic or *metastrategy* for all individual agents, according to which they adapt their action (pure strategy) after having played a game. The rule we specify for the agents is a rule of procedural rationality [1] as described by Simon (1976). It is therefore a behavioral rule [2], see e.g. Camerer (1997). This specific rule is often used in the field of sociology (see e.g. Liebrand and Messick ()) and is based on the idea of reciprocity between players. By specifying the metastrategy we prescribe individual agents how to act in every situation in which they can change their action, and in this way we specify a process by which the prevalence of actions in the population evolves.

[1] Procedural rationality means that people think about their choices before they come to a decision. It focusses on the process or path leading to an outcome. Substantive rationality focusses on the outcome. It supposes that people act as utility maximizers. The common approach in economics is substantive rationality.

[2] According to Camerer (1997), a behavioral rule aims to describe actual behavior, is driven by empirical observations (mostly experiments), and charts a middle course between over-rational equilibrium analyses and under-rational adpative analyses.

This we call a *behavioral evolutionary process* [3].

We are interested in the stable state of the population and in how this stable state is reached. We define a *stable state* as a situation in which the percentages of agents in the population that are playing a certain action is (almost) constant over time for all actions. Note that stable does not mean that all agents stick to the actions they are playing, but is defined on the level of the population as a whole. In the models considered in this paper a stable state always emerges. In this state a lot of socially optimal [4] behavior is observed. However, when applying the model to describe the price selling behavior of producers in an oligopolistic environment with a large number of possible prices, we observe that in the stable state a price war may still occur. When such a price war occurs, a substantial part of the producers in the population lower their prices and the stable state is left. Profits for the producers decrease substantially as a consequence. Within a finite stochastic time after a price war has started the population will again converge towards the same stable state it was in before. Therefore we call the stable state of this model *quasi-stable*. These results are very general, in the sense that a variation of parameter values does not alter the qualitative results. These results can be seen as an alternative explanation for the bubble-crash cycles, as observed by e.g. Smith, Suchanek, and Williams (1988), suggesting that it might not be mere risk aversion that generates the cycles observed by these authors.

The rest of this paper is organized as follows. In the next section we introduce the model and explain the assumptions the model is based on. In section III we briefly discuss the simulation parameters and report the simulation results for the case where there are two actions present in the stage game. Next we look at the simulation results for stage games with more than two possible actions in section IV. In this section we also elaborate on the stability of the results. In section V we show how, in a quasi-stable state, a temporary move away from the stable distribution occurs when coincidentally a trigger point is reached. Finally in section VI we state the conclusions we draw from our analysis.

II. The Model.

In this section we sequentially present the different components of the model.

[3] A behavioral evolutionary process is an evolutionary process with the dynamics specified by a metastrategy based on a behavioral rule.

[4] We consider social optimality on the level of the game. Social optimal thus means that the total payoff from the game is maximal.

II.1. The Social Environment.

In a social environment of economic agents it is often the case that every agent interacts only with a small subset of all agents in the environment. This subset may consist of colleagues, friends, or in case the agents are firms, firms in the same region. The subsets of different agents usually overlap each other. Because all agents can interact with a few other agents and because there is an overlap in the subsets, the influence of agents with whom one does not interact is indirectly present. In these kind of models evolutionary forces can play an important role even in the relatively short term (Ellison (1993, 1995)). We model the social environment and the set of agents with whom one can interact in a specific way in accordance with these observations. Especially we assume interactions to take place sequentially and always to be between two agents.

We model the social environment as a finite $m \times m$ torus in $\mathbb{N}^2$ consisting of m^2 economic agents, being producers of heterogenous goods that are close substitutes, in the sense that the location of the goods of different producers differs. A finite $m \times m$ torus is a structure on which each player is uniquely determined by a location $x = (x_1, x_2)$, with $x_l \in \{1, \ldots, m\}$, $l = 1, 2$ and a player on location x has the 8 players on locations $y = (y_1, y_2) \in \{1, \ldots, m\}^2$, with $y_l = (x_l - 1) \bmod m, x_l, (x_l + 1) \bmod m,\ l = 1, 2$, except $y = x$ as his direct neighbors. Each player interacts only with these direct neighbors. A torus is chosen because it has no borders and therefore all agents are in identical positions. Every agent has a fixed position on the surface of the torus and on every position on the surface of the torus there is exactly one agent located, so there is a one-to-one relation between the set of agents and the surface of the torus. An illustration of this is given in figure II.1.

Interaction between producers takes place in an oligopolistic setting. In the social environment a large number of sequential interactions takes place. A producer is selected from the social environment at random. This producer is called the *subject.* Every interaction consists of playing a game referred to as the *stage game*, which is characterized by oligopolistic price competition. In the stage game, the subject competes for consumers against a fixed environment, consisting of competitors that posted a price for their product. [5]

[5] Although this model is in discrete time, a continuous time version can easily be constructed as follows. Attach a Poisson proces with the same parameter to every agent. Now let every agent be the subject and play the stage game at the times given by her Poisson proces. The simulation model will be exactly the same as the discrete time model presented here.

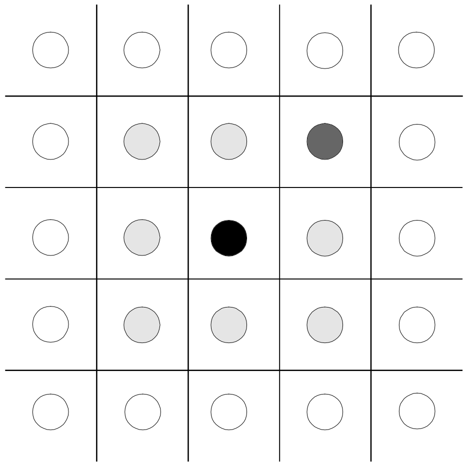

Figure II.1: An illustration of a part of the torus. Every circle represents a producer. The neighbors of the producer indicated by the black circle are shaded.

II.2. The Interaction Structure.

Each producer has a number of consumers who are located very near to her and who normally buy her product. However these consumers compare prices in the neighborhood of the producer. If the price of the subject (selected producer) is high in comparison to that of her neighbors, she will have less customers buying her product. If her price is low in comparison to her neighbors, she will have more customers buying her product. If a consumer chooses not to buy from the subject, it is assumed that he will buy from the neighbor of the subject with the lowest price. If more than one of the neighbors sells for the lowest price, one of them is chosen at random. The rationale for this assumption is that if a consumer is to buy another product than he usually does, he might as well buy the cheapest substitute. This way a producer constantly has to face the competition of her cheapest neighbor. Thus the subject is matched to her cheapest neighbor and plays the stage game. Note that this matching rule implies that a subject will be matched to the most competitive playing producer present amongst her neighbors. After each individual interaction, the subject has the possibility to change the price she set. The next interaction she is involved in, will take place at her new price.

The real world situation we are thinking of when describing this kind of competition is one in which each producer is located in a city. She is the only producer of the good in this city. Inhabitants of this city will normally buy from their local producer. However, if the price of the local producer is too high in comparison with the price of producers in other cities not too far away, the inhabitants might travel to one of the other cities and buy the product there. We assume travel costs back and forth to the local producer to be negligible compared to travel costs to another city. Since (implicit) travel costs are different for all consumers (consumers are heterogenous in travel costs), the decline in demand will be gradual when a producer raises her price. Some of the consumers will buy at a producer in another city very often, other will always buy at their local producer when they buy. This way we have implicitly specified a model of consumer choice behavior, like those that can be found in the location theory in the field of industrial organization (e.g. the linear city model of Hotelling (1929), see also Tirole (1988), all be it that in our model consumers are not uniformly distributed along this interval, but instead groups of consumers are located in the same points as the producers).

Each simulation starts by some random initialization of the environment. Each producer is assigned a random action from the set of possible actions. Also each producer is assigned a random last payoff. The model is ergodic, i.e. the initialization is unimportant. The results do not alter when the initialization is done differently, unless a degenerate initialization like 'all producers initially play the fully cooperative action and get assigned the corresponding payoffs' is chosen. After the initialization the interaction between producers starts.

II.3. The Stage Game.

The stage game consists of a $(k+1) \times (k+1)$ payoff table, characterized by the subject's payoff when she is involved in oligopolistic competition with producers of close substitutes of her own product. When a producer is chosen to play the stage game as subject, she faces a fixed environment, consisting of the prices her neighbors have set. She engages in the game with her own fixed price and observes her resulting payoff. She compares this payoff to the average payoff the neighbors got the last time they played the game as subjects and subsequently updates her action following the metastrategy described below.

Each producer can set $k+1$ different prices, labelled as actions i, $i = 0, \ldots, k$, located at equal distance on the interval between the Bertrand-Nash price p^N, representing the best-reply, given that the competitor will also react optimally, and the (local) monopoly or cartel price p^c. Thus an action i is setting a price equal to $p^N + \frac{i}{k}\left(p^c - p^N\right)$. The price range is restricted to the interval $\left[p^N, p^c\right]$, since this is the interval where the most interesting features occur. At prices below the lower bound p^N, a producer who lowers his price will lower his profit. Above the Nash price, reducing one's price will increase profits. So, the interesting feature that undercutting the price of one's opponent pays off, only holds in the price range above the Nash price. At prices below the upper bound p^c, a price increase by both the subject and his opponent results in a profit increase for both producers. In the price range above p^c profits fall as a result of simultaneous price increase by the subject and his competitor. Thus the range of prices where cooperative behavior, i.e. setting high prices, pays off when the competitor does the same is restricted by the upper bound p^c. We restrict attention to this price range, since this is the kind of reasoning implicit in the update rule 'Win Cooperate, Lose Defect' (WCLD, see below). Relatively high profits induce a tendency towards cooperative behavior with the implicit assumption that when one plays more cooperatively and the competitor does the same, this will result in a better outcome for both players.

The structure of the oligopolistic competition between two competing producers is as simple as possible. From the location theoretic model described above, we have that the number of potential consumers of each producer is equal to a constant $\alpha > 0$, that less consumers will actually buy the product from a producer when the producer's price is higher and that more consumers will buy the subject's product if the price of her cheapest neighbor is higher. Thus the demand function D_i at subject i originating from the consumers located at subject i's location (in the same city as subject i) can be expressed as

$$D_i(p_i, p_{\min}) = \alpha - a_1 p_i + a_2 p_{\min}, \tag{1}$$

where p_i is the price of the subject, $p_{\min}$ is the price of her neighbor with the lowest price and $a_1,\ a_2 > 0$ are parameters. So demand is linear in both prices. Each producer is assumed to have a linear cost function $C(q) = cq$, which only depends on her own production q. In the following we choose the parameters to be $\alpha = 20$, $a_1 = 1$, $a_2 = \frac{1}{2}$ and $c = 1$.

Competitive behavior implies setting the price for one's product equal to the Bertrand-Nash price p^N being the best-reply of a producer when all producers react optimally on the actions of the others and when they all set the same price. The reaction function $p_i = f(p_{\min})$ follows from the profit maximization problem

$$\arg\max_{p_i} \pi^i(p_i, p_{\min}) = D_i(p_i, p_{\min}) \cdot p_i - C(D_i(p_i, p_{\min})). \tag{2}$$

Setting $p_i = p_{\min} = p$ and solving $p = f(p)$ for p with the above parameter values gives the Bertrand-Nash solution $p^N = 14$. The profit of the subject when both she and her competitor set the Bertrand-Nash price is given by $\pi^i(p^N, p^N) = 169.0$.

Cooperative behavior implies that producers will collude instead of competing with each other and thus that the producers are all local monopolists that set the cartel price p^c. This price follows from setting $p_i = p$ for all i and solving the profit maximizing problem

$$\arg\max_{p} \pi^i(p,p) = D_i(p,p) \cdot p - C(D_i(p,p)), \tag{3}$$

which yields $p^c = 20\frac{1}{2}$ for the above parameter values. The profit of the subject when both she and her neighbor act as local monopolists and set the cartel price will be $\pi^i(p^c, p^c) = 190.1$.

When a subject who has set the Nash price competes with a producer who has set the cartel price she will earn a profit of 211.3, whereas a subject who has set the cartel price and competes with a producer who has set the Nash price will earn a profit of 126.8. For a stage game with only two actions, i.e. $k+1=2$, this results in the Prisoner's Dilemma (PD) in table II.1, where the column represents the action of the competitor (C) and the row represents the action the subject (S) is playing.

S \ C	p^c	p^N
p^c	190.1	126.8
p^N	211.3	169.0

Table II.1: The subject's payoff table for the case $k+1=2$.

The payoff table is supermodular, as defined by Topkis (1979) and further explored in Milgrom and Roberts (1990, 1995). In a supermodular payoff table, the two dimensions of the problem (the actions of both players) are complementary, which, in this setting, implies that total profits of the two players taken together is maximal when they both set the cartel price.

II.4. The Metastrategy.

We assume that every producer has (direct) information on her own action and profit and on the last profits of her neighbors only. Note that this means that there is no common knowledge. A producer's current action is determined by her past action and the payoff resulting from her past action relative to the payoffs of her neighbors the last time they played the stage game as subjects. The decision heuristic that is used to update the subject's action is called a *metastrategy* or *update rule.* The metastrategy tells the subject to compare her payoff from the game, π_{self}, with the average payoff her neighbors got from the game the last time they played it as subjects, π_{nbs}.[6] If her payoff is higher than that of her neighbors, she is said to

[6]Note that we implicitly assume that different payoffs can be added up. This is justified in the case that payoffs are profits and players are risk-neutral.

be in a win situation; if her payoff is lower than that of her neighbors she is in a lose situation and if her payoff is exactly equal to the average payoff of her neighbors she is neither in a win nor in a lose situation. This assessment of one's own situation relative to that of others is justified in the psychological and sociological literature by stating that people in general tend to get a good (bad) feeling whenever they think they are doing better (worse) than the other people they can observe. This setting is similar to people having certain *aspiration levels* or comparison levels (see e.g. Thibaut and Kelley (1959)), which they want to achieve. Whenever an agent gets at least (at most) her aspired payoff, she feels good (bad). Here the aspiration level is not a fixed preset value, but instead it is endogenous in the model, as is the case in for instance Palomino and Vega-Redondo (1996). The aspiration level of the subject is the average payoff of her neighbors and it therefore varies.

The metastrategy we use prescribes the subject to change his action as follows. Whenever $\pi_{self} > \pi_{nbs}$, the subject changes her action i to $i+1 \leq k$, so in the next stage game she is involved in, she will play action $i+1$. When $\pi_{self} > \pi_{nbs}$ and the current action of the subject is k, her action remains unchanged. When $\pi_{self} < \pi_{nbs}$, the subject updates her action i to $i-1 \geq 0$. In case $\pi_{self} < \pi_{nbs}$ and the subject's current action is 0, the subject doesn't change her action. When both payoffs are exactly equal, the subject will stick to her current action. This metastrategy is referred to as *'Win Cooperate, Lose Defect'* (WCLD). Underlying this metastrategy is the observation that a producer that observes she is doing well (in monetary terms) will tend to be more cooperative than a producer that observes she is performing rather poorly in comparison with her neighbors. This is an example in which the sociological content of information is a key factor, as is the case in e.g. Roth and Murnighan (1982). It's not about getting a very high payoff, but about doing well compared to a reference group, consisting of agents in a similar situation as one self. At least in the Prisoner's Dilemma game doing well is associated with playing cooperative and in such a game this metastrategy matches positive outcomes with positive outcomes.[7] If a producer gets a bad (good) outcome, she figures her opponent will have behaved badly (nicely) towards her and she will play less (more) cooperative the next time. We therefore consider this metastrategy as a generalized version of Tit-for-Tat, a strategy that turned out to do extremely well in a tournament organized by Axelrod (1987). In Offerman, Sonnemans, and Schram (1996) and in van Lange, de Bruin, Otten, and

[7] For a more detailed treatment of the metastrategy 'Win Cooperate, Lose Defect' in the Prisoner's Dilemma see e.g. Messick and Liebrand (1995).

Joireman (1997) there is also experimental evidence suggesting that this metastrategy is used by a substantial number of people in every-day-life situations. In real life, an agent's actions appear to be very heavily influenced by what she observes about her neighbors. If they are being kind (cooperative), the agent herself will adapt a cooperative action and if they are being nasty (competitive) the agent will adapt the competitive action defect. These kind of 'fairness' assumptions are often discussed and observed in the field of experimental economics (see e.g. Rabin (1993) and the references therein) and sociology (see e.g. Glance and Huberman (1994)).

In short the metastrategy matches positive outcomes with positive outcomes. Being in a win situation enhances pro-social tendencies, as described in Krebs and Miller (1985). They also state that the social behavior of the subject is not restricted to the neighbor the subject last played with. We stress again that this sort of decision heuristic on the level of the individual differs substantially from the usual approach in evolutionary game theory, all be it that using some kind of Tit-for-Tat-like strategy is rather common. We see our model as psychologically driven evolutionary game theory and we will therefore speak of a behavioral evolutionary game theoretic approach.

II.5. The Stable States.

We are interested in the stable state(s) of the model. We define stability on the population level, i.e. a state is stable when the fraction of the population playing a certain action is (almost) constant.

Definition II.1. *A population is in a stable state from time T on, when there exists a vector of population fractions $\overline{x} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k)$, $\sum_{i=0}^{k} \overline{x}_i = 1$, such that*

$$|x_i(t) - \overline{x}_i| < \varepsilon, \ \forall i, \ \forall t > T,$$

where $\varepsilon > 0$ is a fixed small simulation constant and where $x(t) = (x_1(t), x_2(t), \ldots, x_k(t))$, $\sum_{i=0}^{k} x_i(t) = 1$, is the vector of population fractions at time t, with $x_i(t)$ the fraction of the population playing action i, $i = 0, \ldots, k$, at time t.

Note that even when the population is stable, producers constantly change their actions according to the metastrategy. Thus a stable state does not necessarily imply that every producer sticks to her current action forever.

A stable state is present when for all possible actions the number of producers abandoning that action to play another one (outflow) is equal to the number of producers starting to play that particular action (inflow). Note that this criterion flow$_{in}$=flow$_{out}$ for all states, is the stable state criterion of a Markov chain.[8] In the simulation computer program we use the $\text{flow}_{in}=\text{flow}_{out}$ criterion on the level of a simulation run to detect a stable state. A simulation run consists of a fixed number of plays, which is a multiple (the simulation run length) of the number of players in the social environment ($m \times m$) and therefore each player gets to play a certain average number of times (the simulation run length) during one simulation run. An entire simulation consists of a large number of simulation runs. The total number of plays in an entire simulation is equal to *number of simulation runs* $\cdot$ *simulation run length* $\cdot$ *number of players in environment*. Furthermore we implemented a check on the variation in the *average percentage of cooperation*, $100\% \cdot \frac{1}{k} \sum_{i=0}^{k} [x_i(t) \cdot i]$, between one simulation run and the next simulation run, before the $\text{flow}_{in}=\text{flow}_{out}$ criterion is checked. This extra criterion is added for reasons of calculation, so the simulation program doesn't check for the flow criterion unless there is some chance the criterion will actually hold.

III. The Two Action Case.

In this section we report results for the case where the stage game has two actions, i.e. $k+1=2$. In this case the payoff table looks like table II.1, a Prisoner's Dilemma.

III.1. Parameters.

We simulated the above model in a 30×30 social environment, consisting of 900 producers. Furthermore, for each set of parameter values, we performed 300 simulation runs, each having a simulation run length of 50 and we checked for stable states. We varied these parameters in order to perform some comparative statics. Furthermore, we looked at the effect a variation in the size of the neighborhood might have. Instead of picking the cheapest of the eight neighboring producers we let a subject pick the cheapest of the 24 producers that can be reached from the subject in either one or two steps, her two level neighbors. In this case the set of neighbors of a producer in location $x=(x_1,x_2)$, $x_l \in \{1,\ldots,m\}$, $l=1,2$, consists of all players on locations $y=(y_1,y_2) \in \{1,\ldots,m\}^2$, with $y_l = (x_l-2) \bmod m, (x_l-1) \bmod m, x_l, (x_l+1) \bmod m, (x_l+2) \bmod m$, $l=1,2$, except $y=x$. The subject has information concerning last payoffs and current actions of these 24 two level neighbors. We also look at a neighborhood

[8] In fact, the model we consider is a Markov chain, with as state space $(A(t), B(t))$, where $A(t)$ is an $m \times m$ matrix denoting the action of every player at every location at time t and where $B(t)$ is an $m \times m$ matrix denoting the last payoff realized as subject of every player at every location at time t. The number of states in this state space is much too large to calculate an invariant probability measure. Therefore, we focus on the aggregate state space when we talk about stability.

consisting of all 48 three level neighbors, defined in a similar way.

III.2. Results.

The results are summarized in table III.1. In the first column of the table the possible actions

Action	% of population playing
0	49.8
1	50.2

Table III.1: The simulation results for oligopolistic competition against the cheapest neighbor.

of the individual agents are denoted. In the second column the percentage of agents in the population playing this action in the stable state is given.

We see that in a stable state, about half of the producers in the population set the cooperative (local monopoly) price, while the other half set the competitive (Nash) price, a result strikingly different from the 'normal' Nash equilibrium outcome in a competitive environment. In a population in stable state, on average there will be approximately the same number of interactions taking place with (action subject, action competitor) $= (0,1)$ as there are taking place with (action subject, action competitor) $= (1,0)$. The average payoff to the subjects from all these encounters is larger than the payoff of pure competitive play $(0,0)$. Furthermore, around $\frac{1}{4}$ of all interactions takes place with actions $(1,1)$, resulting in a much higher payoff. Thus we can conclude that the expected payoff realized in stable state will be larger than it would have been in the standard outcome of the PD, where every agent plays competitive (sets the Nash price). We conclude that the metastrategy WCLD enhances pro-social behavior. In the next section we will see that the level of cooperation changes when agents are allowed to use more actions than only the two mentioned above.

Varying the parameters did not result in qualitatively different results. Raising the size of the torus m results in higher convergence times, i.e. it takes more time for the population to reach a stable state. Enlarging the size of the neighborhood results in a higher speed of convergence. Varying the length of a simulation run has no effect, when we take into account that a shorter (longer) simulation run results in more (less) variation around the stable distribution $\overline{x}$.constant ε should be larger (smaller).

IV. The Multi Action Case.

In this section we focus on stage games with more than two possible actions, i.e. $k+1 \geq 3$. We simulate the model at the same parameter values as described in section III.1. Except for variations in these parameter values, we focus attention on the influence of varying the values of the parameter k. In the simulation with $k+1 \geq 3$ we encounter quasi-stable states.

Definition IV.1. *The population is in a quasi-stable state, when it satisfies definition II.1 of a stable state, except for recurring small intervals* $[t_1, t_2]$, $t_1, t_2 > T$, *of time.*

A population in a quasi-stable state thus has a vector of population fractions $\overline{x}$, where the state $x(t)$ is very close to most of the time. However, every once in while there are relatively short periods of time when the state $x(t)$ is further than ε away from the vector $\overline{x}$.[9]

IV.1. Results.

In the model with $k+1 \geq 3$ stable states occur only for values $3 \leq k+1 \leq 6$. For these parameter values the features the model exhibits are as described below, except for the fact that the population converges to a stable instead of a quasi-stable state.

For parameter values $k+1 \geq 7$, we find convergence to quasi-stable states. In this case, the results may depend on the size of the neighborhood. For small neighborhoods ($m < 6$ approximately), local effects can easily spread out over the whole population and thus cause global instability. To avoid these effects, we chose to simulate with much larger environment, mostly with $m = 30$. It turns out that the simulation results are not dependent on the parameter value of m, as long as it is not taken too small.

When the number of possible actions producers can take is small, convergence to a quasi-stable state is fast. We will illustrate this by describing the case $k = 10$. When $k = 10$ there is very quick convergence towards a state in which most producers play the actions 8 and 9. Some producers play actions 7 or 10 and only a few producers play an action $i \leq 6$. There are no producers left who play the Nash action 0. When this state is reached, convergence towards the ultimate reported quasi-stable state slows down. This ultimate quasi-stable state consists solely of producers who play actions $6, 7, 8, 9$ or 10. The respective percentages of population playing one of these actions is shown in table IV.1. In small populations (e.g. 10×10) often

[9]Evaluated with the max norm

$$\|x(t) - \overline{x}\|_\infty = \max_i |x_i(t) - \overline{x}_i|.$$

Action	% of population playing
6	0.4
7	8.7
8	28.2
9	38.5
10	24.2

Table IV.1: The simulation results for oligopolistic competition against the cheapest neighbor with $k = 10$ in a 30×30 torus.

only the actions 8 and 9 are played in the quasi-stable state. This whole convergence process takes only a short time. Within a few simulation runs the population is in the quasi-stable state.

After the quasi-stable state is reached the population stays near the state $\overline{x}$ for a stochastic period, after which there is a move away from state $\overline{x}$. The rare state of the population $x(t)$ that causes this phenomenon is called a *trigger point*. In section V we elaborate further on the trigger point phenomenon. When the state $\overline{x}$ is left, there suddenly appears a lot more competitive behavior in the population. After a little while however, the state of the population starts to converge again and to the same state $\overline{x}$ as before. The behavior of the population immediately after the trigger event has occurred can be regarded as a *price war*. A producer feels forced to lower his price below the 'stable' level. As a reaction more consumers will buy his product and other producers will more frequently be in a lose situation when they play the stage game with this producer. As a consequence, they will also lower their price, thereby starting a downward spiral of the prices in a subset of the population. These effects are very clearly visible in small populations, where the entire population is affected by such a price war. In larger populations, the effects of a price war are only local. After a price war has lasted for a stochastic time (longer in a small population, shorter in a large population) the affected producers all get a lower profit than they used to have and by playing against similar producers with low profits, they will more often realize a higher profit than the average profit of their neighbors. Direct result of this is that these producers will again begin to display more cooperative behavior. The convergence process towards the state $\overline{x}$ has started again. Note that the state with low prices, which occurs at a price war, is not stable.

Although the stable state depends on the parameter k, the features described above do not change for the intermediate range of k's, i.e. k not too small and not too large. As can be seen in section III.2, for $k = 1$ the average percentage of cooperation in the population is approximately

50%. In this case price-wars do not occur. The average percentage of cooperation in the population in the quasi-stable state, rises monotonous with k, up to about 97% when $k = 35$. An illustration of how this rise takes place is given for $1 \leq k \leq 35$ in figure IV.1 and a few typical encountered stable states are reported in appendix A. Since higher values of k can be interpreted as a weaker response of the agents to satisfaction or dissatisfaction, we see that a weaker response of an agent that updates leads to a higher ultimate degree of cooperation in the population.

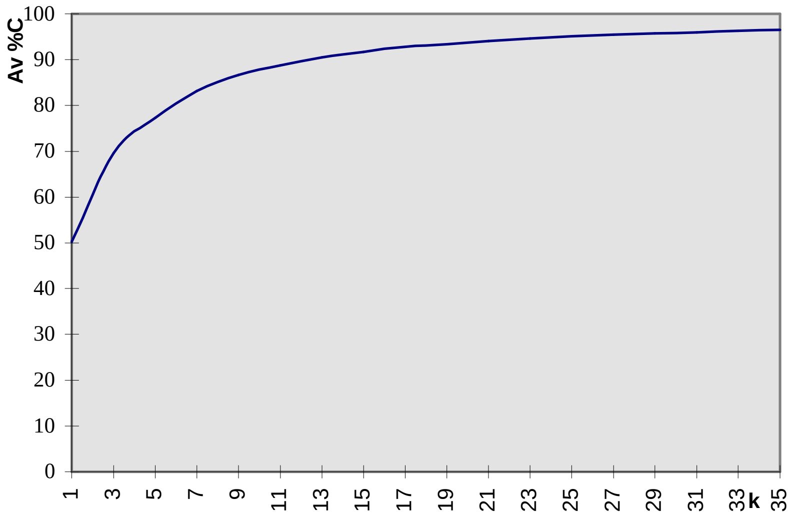

Figure IV.1: Graph of the relation between k and the average percentage of cooperation in the population in the quasi-stable state.

For parameter values above $k = 5$ we observe price wars taking place.[10] Interesting with respect to the parameter k is furthermore that for increasing k (but still $k \leq 35$), convergence towards the quasi-stable state of the population still evolves essentially according to the same pattern we observed in a setup with k small. Still there is first convergence to a state in which most producers play one of the higher labelled actions (but this state is not the quasi-stable state yet) and after this initial, relatively fast convergence, a slower process towards a quasi-stable state starts. In all observed quasi-stable states, almost all producers play a high labelled

[10]When k becomes larger than 35, the average percentage of cooperation in the population does not increase further. More strikingly, there may arise equilibria in which a lot of competitive behavior is observed (defect equilibria). The sudden appearance of defect equilibria for values $k > 35$ is very surprising. The more because it seems only to be present in a setting where producers have only a small number of neighbors they can interact with. We will work on the explanation of this phenomenon in future research.

action, corresponding to setting a price in the upper quarter of the interval $\left[p^N, p^c\right]$.

Again, we also varied the neighborhoodsize when the stage game has more than two actions. We looked at the effects of extending the set of neighbors to incorporate all 24 two level neighbors and to the effect of further extending the neighborhood to incorporate all 48 three level neighbors. In general, for most values of k the quasi-stable state that is reached with large neighborhoods does not differ much from the quasi-stable state that is reached with smaller neighborhoods. What does differ is the variation across the observations. This turns out to be less when the neighborhood becomes larger. A result that is intuitively appealing, since a wider scope for the individual makes the persistence of small areas with different behavior much more difficult and therefore should guarantee that there is less variation in actions across the population.

It turns out that when the neighborhood becomes larger, the speed of convergence rises. This result seems to be in contrast with Ellison's (1993, 1995) result on convergence rates in models with local interaction and Darwinian dynamics. He concludes that convergence gets slower when the size of the neighborhood increases. A possible explanation might be given by the following. The main effect in Ellison's model with Darwinian dynamics is that a larger neighborhood size implies that it is harder for a mutant action to gain foothold in a population in equilibrium. This effect is of lesser importance in our model, since the essential mechanism behind our result has nothing to do with the presence of mutants. The effect described above, that a larger neighborhood causes a tendency towards more homogeneous actions in the population seems to be more important in our model. This effect causes the initial convergence towards a state nearby the quasi-stable state to be swift, thus resulting in faster convergence altogether.

V. The Trigger Points.

We already mentioned the existence of trigger points when the population is in a quasi-stable state. In this section we want to give some insight in how a trigger state $x(t)$ may evolve. We will illustrate this by means of five figures of the same 3×4 subsection of the population for the parameter value $k = 10$. As we already mentioned in section IV.1, a combination of a small population and a parameter value of $k = 10$ can result in a stable state consisting solely of producers who play the actions 8 or 9. In the subsequent analysis such a situation will be our point of departure. We show that a specific outcome of the random process selecting producers, induces one of the producers to lower her price below any price that is played in her neighborhood. This can be viewed as a spontaneous mutation that arises endogenous in the model, which is different from the models by Young (1993), Young and Foster (1991), Kandori, Mailath, and Rob (1993) and Ellison (1993, 1995) where spontaneous mutations are exogenous features that change the outcome of the models in an essential way.

The subject's payoffs of the interaction between the subject (S) and a competitor (C), when they are both playing one of the actions 8 or 9 are given by [11]

S\C	8	9
8	189.28	195.20
9	183.79	189.91

.

Figure V.1 gives the initial state of a small part of the social environment in which all producers play either the action 8 or 9. Each box represents a producer and we label the producers as (i, j), $i = 1, 2, 3$, $j = 1, \ldots, 4$, where the label $(1, 1)$ indicates the upper-left producer. Every producer's last played action is denoted in the upper-left corner of the box, while in the bottom-right corner the last realized payoff is denoted.[12] The situation as depicted, in which 11 out of

9 189	9 189	9 189	9 189
9 189	9 189	8 184	9 189
9 189	9 189	9 189	9 189

Figure V.1: A part of the social environment: The initial situation.

the 12 producers have a last payoff of 189.28, can arise when they all played action 8 in their last game as a subject and that their competitors were also producers playing action 8. The producers who were in a win situation changed their action into 9 after having played. The producer at position $(2, 3)$ has been playing 9 against an 8 playing opponent, realized a payoff of 183.79, which was lower than the average payoff of her neighbors and therefore changed her

[11] Note that this is a part of the full 11×11 payoff table.

[12] In the picture we present only rounded values of the last payoffs. The mathematics however are done with the unrounded values of the last payoffs.

action into 8. We will now describe one particular series of random events that leads producer $(2,2)$ to adapt action 7, which is at the moment not present in (this part of) the population.

We let the random mechanism select producer $(2,3)$. All neighbors of this subject are identical with respect to the price they set. The subject plays against one of these neighbors and realizes a payoff of 195.20. The subject is clearly in a win situation with this payoff and therefore she updates her action to 9. We are now in the situation depicted in figure V.2, which is the trigger point. Suppose producer $(2,2)$ is selected. Since all her neighbors play action 9,

9 189	9 189	9 189	9 189
9 189	9 189	9 195	9 189
9 189	9 189	9 189	9 189

Figure V.2: A part of the social environment: Step 1.

she will play against an arbitrary neighbor of hers and realize a payoff equal to 189.91, which is lower than the average payoff of her neighbors. Subsequently she will change her action to 8 and the situation in figure V.3 arises. Now suppose the random mechanism selects producer

9 189	9 189	9 189	9 189
9 189	8 190	9 195	9 189
9 189	9 189	9 189	9 189

Figure V.3: A part of the social environment: Step 2.

$(1,3)$, although any of the neighbors of producer $(2,2)$, except producer $(2,3)$, would do. This subject will play against a competitor using action 8 (not necessarily producer $(2,2)$) and gets a payoff of 183.79. We assert she loses and updates to action 8. We have now arrived in the situation of figure V.4.

Now the random mechanism again selects producer $(2,2)$. This producer competes with

9 189	9 189	8 184	9 189
9 189	8 190	9 195	9 189
9 189	9 189	9 189	9 189

Figure V.4: A part of the social environment: Step 3.

her neighbor $(1,3)$ who plays action 8 and therefore producer $(2,2)$ will get a payoff of 189.28. This again puts her in a lose situation and therefore she changes her action to 7 as depicted in figure V.5.

9 189	9 189	8 184	9 189
9 189	7 189	9 195	9 189
9 189	9 189	9 189	9 189

Figure V.5: A part of the social environment: Final state.

Now the population is no longer in a stable situation, instead it has entered one of the small intervals $[t_1, t_2]$, mentioned in definition IV.1 of a quasi-stable state. Since there is one 7 playing producer in the population, other producers who play against this producer will be in the lose situation and will therefore be forced to lower their prices. The population as a whole will move in the direction away from the state $\overline{x}$.

After some stochastic time however, there will be enough producers with a low last payoff to ensure that some producers will realize a payoff higher than the average payoff of their neighbors and thus will be in win situations again. The trend away from the state $\overline{x}$ is reversed and there is again convergence towards the same state $\overline{x}$ as before.

The essential feature of the example of a trigger point is that one producer gets to play twice in very short succession, i.e. before any of his neighbors has played more than once. The events as described above have a very small probability of happening, but since there are many situations alike in which a trigger point is reached, the overall probability of reaching a trigger point is large enough to observe the effects very clearly.

VI. Conclusions.

In this paper we have shown that cooperative behavior can evolve in a social environment consisting of producers who play Bertrand price competition, when the producers follow behavioral rules that stem from sociology and when interaction is local. Cooperative behavior evolves for a wide range of parameter values we considered, so we can state that this result is robust. The average percentage of cooperative behavior in the population that emerges is fairly high. The weaker the response (the price adjustment) when a player updates, the higher the ultimate degree of cooperation that emerges. The percentage of cooperative behavior varies, but stays within the bounds of $[90\%, 95\%]$. Therefore we conclude that our behavioral model offers an explanation for the emergence of highly cooperative behavior in a population. This kind of behavior is regularly observed in experiments.

In an environment with a high degree of cooperative behavior, we observe the emergence of price wars. These wars cause a temporary fall in prices and have global (in small environments) or local (in large environments) effects. The behavioral evolutionary model can therefore be useful to explain the emergence of the phenomenon 'price war' or price crash-bubble cycles.

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A. Stable States.

Here some quasi-stable states of model in section IV.1 are reported to in detail for different values of $k+1$, the number of actions in the stage game. In the top row are the possible actions. Remember that an action i, $i = 0, \ldots, k$, is setting a price $p^N + \frac{i}{k}\left(p^c - p^N\right)$. In the bottom row are for every action the percentage of the population playing that particular action. In the last column the average percentage of cooperation in the population is reported. This is simply the average of the actions weighted by the percentage of the agents playing that action in the stable state.

$k+1 = 10:$

Action	0	1	2	3	4	5	6	7	8	9	Av %C
%Playing	0.0	0.0	0.0	0.0	0.0	0.2	7.6	28.6	39.7	23.9	86.6

$k+1 = 20:$

Action	0	1	2	3	4	5	6	7	8	9	10
%Playing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Action	11	12	13	14	15	16	17	18	19	Av %C	
%Playing	0.0	0.0	0.0	0.0	0.8	9.1	28.6	38.3	23.2	93.4	

$k+1 = 30:$

Action	0	1	2	3	4	5	6	7	8	8	10
%Playing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Action	11	12	13	14	15	16	17	18	19	20	21
%Playing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Action	22	23	24	25	26	27	28	29	Av%C		
%Playing	0.0	0.0	0.0	0.6	9.0	28.2	38.2	24.0	95.7		